DYNAMIC ANALYSIS OF TRACTION FORCES IN A TWO-ROPE SYSTEM: INFLUENCE OF LOADING RATE AND INERTIA

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Abstract: *This paper examines the physical behavior of a system subjected to slow and sudden traction using an experimental device consisting of a suspended mass connected by two identical ropes. The study highlights the conditions under which either the upper or the lower rope fails, depending on the pulling regime. When the lower rope is pulled abruptly, its tension increases rapidly and exceeds the breaking limit, while the upper rope remains intact due to the inertia of the mass. In contrast, a gradual pull leads to a quasi-static state in which the upper rope supports both the applied force and the weight, causing it to break first. The analysis is extended by considering elastic properties and introducing a non-dimensional parameter related to pulling speed, allowing the identification of a critical regime. The results emphasize the combined influence of inertia, force magnitude, and loading rate on traction dynamics.*

Keywords: *traction force, inertia, rope tension, dynamic loading, critical speed, experimental device*

1. INTRODUCTION

Traction force is a fundamental concept in mechanics, defined as the force that tends to stretch or elongate a body along a given direction. It plays a crucial role in a wide range of engineering and physical systems, from vehicle motion and cable-driven mechanisms to structural elements subjected to tensile loading. Understanding how traction forces are transmitted and distributed within a system is essential for predicting mechanical behavior and preventing failure.

A particularly interesting and counterintuitive phenomenon appears in systems involving a suspended mass connected by two ropes: one attached to a fixed support and the other subjected to an external pulling force. Depending on how the lower rope is pulled, either slowly or abruptly, different failure modes can occur. A sudden pull typically causes the lower rope to break, while a gradual pull leads to failure in the upper rope. Although this experiment is simple and widely known in physics education, its explanation involves important concepts such as inertia, dynamic loading, and force distribution.

This paper aims to analyze this phenomenon both experimentally and theoretically, using a dedicated device that allows controlled application of traction. The study focuses on clarifying the role of inertia and identifying the conditions under which the transition between failure modes occurs, providing a clearer understanding of traction behavior in dynamic systems.

2. THE PHENOMENON IN STUDY

The system considered consists of a rigid body of mass m, suspended vertically by an upper rope attached to a fixed support, while a second rope with similar mechanical properties is connected to the lower part of the body. An external force is applied to the lower rope, either gradually or abruptly, in order to investigate the response of the system.

A counterintuitive effect is observed: a sudden pull generally leads to the failure of the lower rope, whereas a slow and progressive pull causes the upper rope to break first. This behavior cannot be explained using only static equilibrium considerations, as it depends strongly on the rate of loading and the dynamic response of the system.

The key factor governing this phenomenon is the inertia of the suspended mass. In the case of a rapid pull, the applied force produces a sharp increase in tension in the lower rope, while the mass does not have sufficient time to respond through significant motion. As a result, the lower rope reaches its breaking limit first. Conversely, under slow loading, the system evolves through a sequence of quasi-static states, and the upper rope supports both the applied force and the gravitational load, leading to its failure.

To provide a deeper understanding of this behavior, it is necessary to develop a mathematical model that describes the time evolution of the tensions in both ropes. In the following section, the system is analyzed by applying Newton's second law and considering the elastic properties of the ropes, which allows the identification of a critical regime associated with the pulling speed and the transition between the two failure modes.

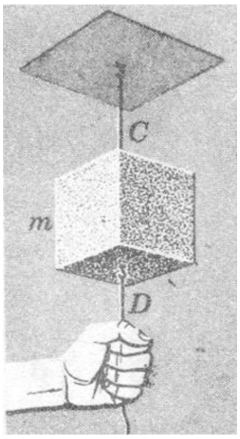

Fig. 1. Experimental setup for the study of traction behavior

3. EXPLANATION OF THE PHENOMENON

If the lower rope is tugged rather abruptly, it breaks, and the one from above doesn't, so that the body still stays suspended from the ceiling. Conversely, if we repeat the experiment from the same initial state, pulling very slowly but gradually of the lower rope, we notice that the rope from above is the one that breaks first, and the suspended body falls to the floor[1].

By this very simple experiment, which can be done by anyone, the role of "inertia" of the suspended body can be highlighted. Inertia makes the upper rope not to break when the lower on is suddenly jolted. The jerk (short as duration) makes the tension of the lower rope increase suddenly up to very high values. The high inertia of the suspended body makes the acceleration not to increase abruptly. The lower rope rapidly overcomes the "break value". In the other case, when traction is slow, tension in the upper rope is permanently higher than that of the lower rope, since weight also matters (the block's weight)[2]. In this case, the "break value" of the tension is reached more quickly in the upper rope than in the lower rope.

This primary analysis should be continued and brought further. If the ropes are extensible, irrespective of the not so high intensity of the traction (stretching), the massive body becomes oscillating vertically, which leads to a modulation of the tension between the two ropes.
Depending on the strength of the tension (of the lower rope), and the elastic properties of the ropes, we cannot firmly, definitely answer to the question "which rope breaks first?"
(since modulation can pass the higher value of the tension upwards or downwards!).
We shall next carefully analyze the case of traction of constant *V* speed, and especially, the field of the "critical speed", the speed that corresponds to going from one kind of break to the other kind of break[3-5].

We admit that the ropes of the experiment [$(C) \equiv (1)$ - the upper; $(D) \equiv (2)$ - the lower] have the same Young module (*E*), but being also dependent on the length, their constants will be differently considered[$k_1 \neq k_2$]. Let F_{max} be the break tension, common to the two ropes. We admit that the Law od Hooke is valid.
L_i id the un-tensioned lengths of the ropes, and ΔL_i, $i = 1; 2$, the corresponding elongations. We can write:

$$F_i = k_i \Delta L_i \quad F_I \lhd F_{\max}, \tag{1}$$

Let us look at the next figure (Fig.2). The lower rope is vertically pulled down, with constant speed *V*. At $t>0$ the length of the lower rope is $L_2 - x + Vt$, so that the force in this rope will be:

$$F_2 = k_2 \left[\left(L_2 - x + Vt \right) - L_2 \right] = k_2 \left(Vt - x \right), \tag{2}$$

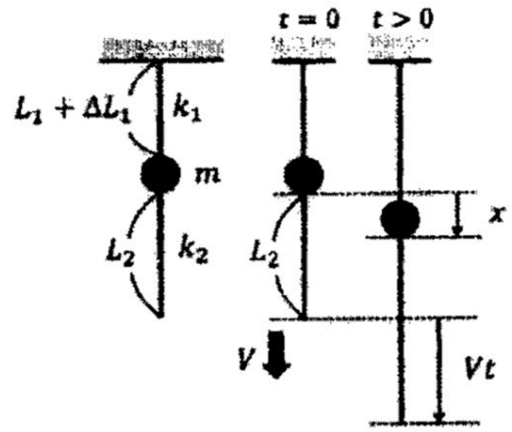

Fig. 2. Scheme of the experiment

The force in the first rope is:

$$F_1 = k_1 x + mg, \tag{3}$$

The movement of the body is described by Newton's second Law $m\ddot{x} = mg + F_2 - F_1$ which, in the end will have the form:

$$\ddot{x} = (k_2 / m)Vt - [(k_1 + k_2)/m]x \quad , \tag{4}$$

Noted:

$$\omega \equiv \sqrt{(1/m)(k_1 + k_2)} \quad , \tag{5}$$

And with the initial conditions $x = \dot{x} = 0$ for $t = 0$, we have the general solution:

$$x(t) = \frac{Vk_2 t}{m\omega^2} - \frac{Vk_2}{m\omega^3}\sin \omega t \quad , \tag{6}$$

Using dependent (6), the forces in the ropes get the form:

$$F_1(t) = mg\frac{k_1 k_2 Vt}{m\omega^2} - \frac{k_1 k_2 V}{m\omega^3}\sin \omega t \quad , \tag{7}$$

respectively

$$F_2(t) = k_2 Vt - \frac{k_2^2 Vt}{m\omega^2} - \frac{k_2^2 V}{m\omega^3}\sin \omega t = ... = \frac{Vk_2^2}{m\omega^3}\sin \omega t + \frac{k_1 k_2}{k_1 + k_2}Vt \tag{8}$$

We now introduce a non-dimensional value (speed):

$$v^* = \frac{Vk_2}{mg\omega} \quad , \tag{9}$$

As well as notations:

$$P_1 = \frac{k_1 g}{\omega^2}, \quad P_2 = \frac{k_2 g}{\omega^2}, \quad \frac{P_i}{mg} = \frac{k_i}{k_1 + k_2}, \quad \frac{P_1 + P_2}{mg} = 1 \quad , \tag{10}$$

With the new notations, the solutions for the forces of tension in the ropes become:

$$F_1 = v^* P_1(\omega t - \sin \omega t) + mg \quad , \tag{11}$$

$$F_2 = v^* P_1 \omega t + v^* P_2 \omega t \quad , \tag{12}$$

Or, in the "normalized" (non-dimensional) form:

$$F_1 / mg = v^* P_1(\omega t - \sin \omega t)/mg + 1 \quad , \tag{13}$$

$$F_2 / mg = (v^* P_1 / mg)[\omega t + (P_2 / P_1)\sin \omega t] \quad , \tag{14}$$

Let us assume that, at a certain "critical" moment in time, let's say $t = t_c$, tensions in the two ropes become equal, that is, $F_1 = F_2$. The equations from above give:

$$\sin \omega t_c = (1/v^*)[mg/(P_1 + P_2)] = 1/v^* \quad , \tag{15}$$

Thus, if $v^*>1$, equation (14) has a real solution and there is at least one moment t, when the fraction F_1/F_2 goes from improper to proper values, or vice versa. We can say that relation $v^* \geq 1$ id a necessary condition for the regime of going from "finely pull" to "jolt"(or vice versa) to exist.

When speed V is low (fine pull), the forces are in the relation $F_1 > F_2$ for any t, first the upper rope will break (when value F_{max} is reached.

This scenario disappears if we pull the rope so that $v > 1$. In certain time intervals, the order relation is $F_1 < F_2$ which means the possibility for the lower rope to break first if F_2, gets to be equal to, and tends to exceed value F_{max}. In conclusion, depending on the concrete values of F_{max} and v^* first either the upper, or the lower rope can break.

4. CONCLUSIONS

Pulling the rope slowly, we exert a tension in the rope above and under the weight. Due to the mass of the weight, the tension above the weight is much higher than under it. The rope breaks where the tension is the highest. When a sudden tug is exerted above the rope, the inertia of the weight keeps the tension under the weight. Although there is a certain tension above the weight, compared to the tension under the weight, the tension in the latter is still higher, and the rope breaks under the weight.

The tension in the upper rope is totally given by the distance between the ball and the support! Think of this rope as being a very rigid arch.

To increase this tension, the ball should move downwards. Only a little, not much, but it has to move to stretch the rope.

If one pulls slowly, the ball has time to move and it does, then the upper row breaks.

If one pulls quickly, the ball has no time to move and stretch the upper rope until the point of breaking before the lower rope would break. The additional tension in the lower rope bestows a net the ball, but this makes it to accelerate only, not to move instantaneously. In the short time before the lower rope breaks, it accumulates a little speed, but it is not a net movement.

In conclusion, depending on the concrete values of F_{max} and v^* first either the upper rope may break, or the lower one.

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