KINEMATIC ANALYSIS OF A PLANAR MECHANISM BASED ON CIRCLE INVERSION FOR MOTION TRANSFORMATION

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Abstract: *The paper presents aspects regarding a device (mechanism) that transforms the circular motion of a point of a device into the linear motion of another point also belonging to the same device and the geometrical transformation of circle inversion.*

Keywords: Device, inversion, circular, linear motion

1. INTRODUCTION

A mechanism is a system of components working together to transmit force, motion, or energy, transforming inputs into desired outputs. Similarly, a mechanism refers to the detailed mode in which something works (internal mechanisms of a process in science (for example, a chemical reaction), nature or even administration).
An inversion in mathematics is a bijective function that transforms a figure by changing point by point the positions of the figure's points. Inversion is part of the category of geometric transformations. A geometric transformation converts a geometric figure into another.
There are several types of geometric transformations. In mathematics, a geometric transformation is a notion similar to an algebraic operation, where the operands are geometric elements. It is a bijection of a set on itself (or on another such set), with important geometric characteristics.

2. MECHANISM

The mechanism represented in Fig. 1 is made up of the identical rods *AB* and *AC*, jointed in point *A* with a hinge (juncture), and the other four rods making up the rhombus *MBNC*, equipped with hinges (junctures) in points *B*, *M*, *C,* and *N*. While the mechanism pivots(rotates) around fixed point *A*, the joint in point *M* can slide only on a circular guide, represented in the drawing by the dotted circle.

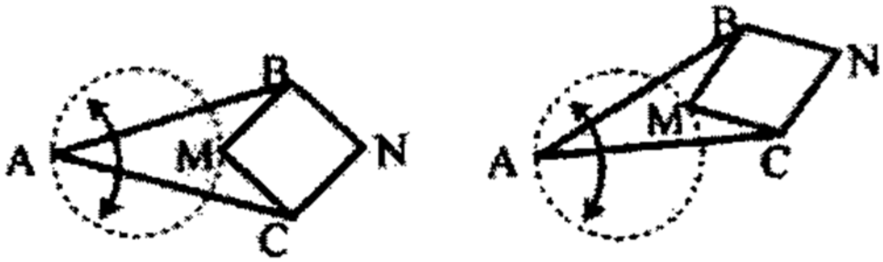
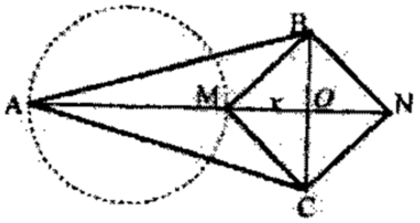
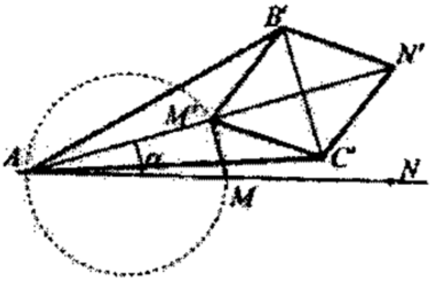


Fig. 1. Presentation of the mechanism

The dimensions of rods AB = AC = L, MB=MC = NC= NB = l as well as radius R of the dotted circle being known, determine the locus of point N, when the mechanism pivots(rotates) around the fixed point A (as it is schematically shown in Fig. 2).





a) b)

Fig. 2. Rotation of the mechanism

Next, we shall give the solution for determining the locus.
We introduce the notation x = OM=ON (Fig. 2, a). With Pitagora's theorem, we can express segment BO in two various modes:

$$OB^2 = AB^2 - AO^2 = MB^2 - MO^2, \tag{1}$$

We thus have the equality:

$$L^2 - (2R + x)^2 = l^2 - x^2, \tag{2}$$

whence:

$$L^2 - 4R^2 - 4Rx - x^2 = l^2 - x^2 \Rightarrow x = (L^2 - l^2)/4R - R \tag{3}$$

then,

$$AN = 2(R + x) = (L^2 - l^2)/2R, \tag{4}$$

When the mechanism rotates with angle α around the fixed point A, point M goes to M' (found on the circle of radius R) and AM' = AM cosα. We use a similar reasoning as above. We introduce the notation $y = OM' = ON'$ (Fig. 2, b). With Pitagora's theorem, we can express the segment $B'O$ in two different modes:

$$B'O^2 = AB^2 - AO^2 = M'B'^2 - M'O^2, \tag{5}$$

Thus, we have the equality:

$$L^2 - \left(2R\cos\alpha + y\right)^2 = l^2 - y^2 \;, \tag{6}$$

whence:

$$L^2 - 4R^2\cos^2\alpha - 4Ry\cos\alpha - y^2 = l^2 - y^2 \Rightarrow x = \left(L^2 - l^2\right)/\left(4R\cos\alpha\right) - R\cos\alpha \tag{7}$$

then,

$$AN' = 2\left(R\cos\alpha + y\right) = \cancel{2R\cos\alpha} + 2\frac{L^2 - l^2}{4R\cos\alpha} \cancel{-2R\cos\alpha} = \frac{L^2 - l^2}{2R\cos\alpha} \;, \tag{8}$$

And from equation (4) we have: $AN = 2\left(R + x\right) = \left(L^2 - l^2\right)/2R$, whence from equation (8) results:

$$AN' = \left(L^2 - l^2\right)/\left(2R\cos\alpha\right) = AN/\cos\alpha \;, \tag{9}$$

Hence, *AN = AN'cosα*, which means that point *N'* is found on the perpendicular erected in *N* on line *AMN*. This device, known as *Lipkin-Peaucellier* device, (invented in 1864), transforms the circular motion (of *M*) into linear motion (of *N*). It is a plane mechanism, the first of this kind capable of transforming circular motion in perfect rectilinear motion (straight line), and vice versa, without additional guides, based on the mathematical concept of circle inversion (a rhomboid and two longer arms), pivoting around a fixed point. It has six hinged arms: two long arms (usually of similar length), and four shorter arms, making up a rhomboid (or parallelogram). The long arms are hinged in a fixed point, and at their ends they join the opposite peaks of the rhomboid. When an arm of the rhomboid moves along a circumference, the other elements are thus configurated, so that another point fixed on the rhomboid would describe a perfect straight line.

3. CIRCLE INVERSION

Circle inversion in geometry is a transformation mapping the points of a circle (or of the plane) on other points, keeping certain properties. Inversion is done in relation with a fixed point, called the inversion pole (*O*), and a real number, the inversion radius (*k*). If the inversion pole *O* is outside the circle or on it, the circle transforms in another circle. If the circle goes through the inversion pole (*O*), its transformation is a line (perpendicular on the radius that *O*, if applicable). If the circle does not include pole *O* and does not include (*P*), it transforms in a new circle (*C'*) the center of which is found on the line uniting *O* with the center of the initial circle (*C*).

Or, starting from the following results:

Let *C(O, r*) be a circle in a plane, with the center in the fixed point *O* and radius *r*, then:

Whichever points $A, B, A', B' \in C(O,r)$ would be, in the case that lines *AB* and $A'B'$ intersect in a point *P*, the following equality happens:

$$PA \cdot PB = PA' \cdot PB', \tag{10}$$

If a variable secant goes through a fixed point *P* and intersects a circle *C(O,r)* in points *A* and *B*, then the product:

$$PA \cdot PB = ct, \tag{11}$$

is constant.

Starting from the fact that inversion is a geometric transformation where each point (*P*) (except pole (*O*)) is transformed in a point (P') is found on the same radius from (*O*), so that the product of distances $OP \cdot OP' = k^2$ (where *k* is the inversion constant), we shall consider point *A* the inversion pole, and points *M* and M' found on circle $C(O_1, R)$, and points *N* and N' the transforms of points *M* and, respectively M' (fig. 3). Considering points *N* and N' the transforms of points *M* and M', and taking into consideration the relationships from above, we can write the following products of distances:

$$AM \cdot AN = 2R \cdot \frac{L^2 - l^2}{2R} = L^2 - l^2, \tag{12}$$

$$AM' \cdot AN' = 2R\cos\alpha \cdot \frac{L^2 - l^2}{2R\cos\alpha} = L^2 - l^2, \tag{13}$$

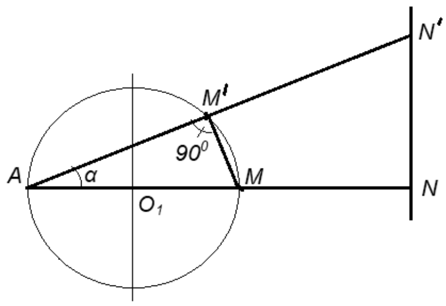


Fig. 3. Transformation of points *M* and M' as to pole *A*

$$AM \cdot AN = AM' \cdot AN' = L^2 - l^2, \tag{14}$$

From equation (14) the fractions (equality) result:

$$\frac{AM}{AN'} = \frac{AM'}{AN}, \tag{15}$$

From Fig. 3, one can notice that se AM' and AM are the sides of the rectangular triangle $AM'M$ ($\sphericalangle AM'M = 90^0$, under a semicircle written), and AM side is opposed to the angle of 90^0. Also, from Fig. 3 one can see that AN' and AN are sides of triangle ANN'. Based on the fraction in equation (15) , it results that triangles $AM'M$ and ANN' are similar. Thus triangle ANN' is also rectangular, and side AN', as per the similarity relation (AM / AN') opposes $\sphericalangle ANN'$ that should be of 90^0. Thus points N and N' are found on the same line, and we have:

$$\frac{AM}{AN'} = \frac{AM'}{AN} = \frac{MM'}{NN'}, \qquad (16)3$$

Thus, the image of the circle that goes through the inversion pole is a line.

4. CONCLUSIONS

The mechanism is the first plane mechanism capable of transforming a circular motion in a perfect rectilinear motion. The mechanism functions based on the concept of inversion as to a circle (circle inversions).

5. REFERENCES

[1]. **Uliu, Fl.,** *Teme experimentale și probleme de fizică aplicată. Pentru pregătirea Olimpiadelor și Concursurilor Școlare de Fizică*, Ediția a 6-a, Editura Emia, Deva, 2020;

[2]. *** https://www.scribd.com/doc/224676094/Capitole-Speciale-de-Geometrie-Pentru-Profesori-Camelia-Frigioiu

[3]. *** https://www.scribd.com/document/445058589/03a-Matematica-3-Elemente-de-transformari-geometrice-opti-pdf

[4]. **Kreyszig, E.,** *Advanced Engineering Mathematics*, Ediția a 10-a, John Wiley & Sons, New York, 2011;

[5]. **Uicker, J.J., Pennock, G.R., Shigley, J.E.,** *Theory of Machines and Mechanisms*, Ediția a 5-a, Oxford University Press, New York, 2017;

[6]. **Wilson, C.E., Sadler, J.P.,** *Kinematics and Dynamics of Machinery*, Ediția a 3-a, Pearson Education, Upper Saddle River, 2013;

[7]. **Sclater, N., Chironis, N.P.,** *Mechanisms and Mechanical Devices Sourcebook*, Ediția a 5-a, McGraw-Hill, New York, 2011;

[8]. **Stillwell, J.,** *Geometry of Surfaces*, Springer, New York, 2010;

[9]. **Needham, T**., *Visual Complex Analysis*, Oxford University Press, Oxford, 2014;

[10]. **Itu, R.B., Șoica, A., Marc, B.-I**., Applications of Lami's Theorem to Coplanar Concurrent Forces in Statics, Annals of the University of Petroșani, Mechanical Engineering, Vol. 27, Petroșani, 2025, pp. 91–100;