MATHEMATICAL MODELING AND SIMULATION OF VIBRATIONS IN MINING MECHANICAL SYSTEMS FOR RELIABILITY AND PERFORMANCE OPTIMIZATION

Assistant, Alexandra SOICA, University of Petrosani, Department of Mechanical, Industrial and Transport Engineering**,** alexandrasoica@upet.ro
Assistant, Cristina PUPAZA, University of Petrosani, Department of Mechanical, Industrial and Transport Engineering, cristinasuciu@upet.ro
Lecturer, Mihaela TOMESCU, University of Petrosani, Department of Mechanical, Industrial and Transport Engineering, mihaelatomescu@upet.ro
Lecturer, Susana APOSTU, University of Petrosani, Department of Mechanical, Industrial and Transport Engineering, susanaapostu@upet.ro

Abstract: *Vibrations are a common phenomenon in mining mechanical systems and have a significant impact on the performance, reliability, and service life of equipment. In mining applications, such as belt conveyors, crushers, drilling machines, and hoisting systems, excessive vibrations can lead to premature wear, structural degradation, and unexpected failures, affecting both operational efficiency and safety. In this context, mathematical modeling of vibrations represents an essential tool for understanding the dynamic behavior of mining equipment and for improving system performance. This paper presents the fundamental principles of mathematical modeling applied to mechanical vibrations, focusing on the analysis of dynamic behavior in mining systems under real operating conditions. Mathematical models based on single-degree-of-freedom and multi-degree-of-freedom systems are developed using differential equations to describe vibration motion. Key parameters such as equivalent mass, system stiffness, damping coefficient, and their influence on the dynamic response of mining equipment are also examined. The study further investigates methods for analyzing free and forced vibrations, as well as the effects of resonance on the stability and safe operation of mining machinery. Mathematical modeling enables the simulation of system behavior under various working conditions, allowing the identification of critical parameters and the development of effective vibration reduction solutions. The results highlight the importance of mathematical modeling in the design, monitoring, and optimization of mining mechanical systems. The application of these methods contributes to improved operational efficiency, reduced maintenance costs, and enhanced safety in mining operations.*

Key words: Damping; Dynamic analysis; Mathematical modeling; Mining equipment; Reliability; Vibrations

1. INTRODUCTION

Mechanical vibrations are inherent phenomena in industrial systems and play a critical role in the performance and reliability of engineering equipment. In the mining industry, where machinery operates under harsh conditions, high loads, and continuous regimes, vibrations are particularly significant. Equipment such as belt conveyors, crushers, drilling machines, and hoisting systems are frequently subjected to dynamic forces that can generate complex vibration patterns, affecting both structural integrity and operational stability.

Excessive vibrations in mining mechanical systems may lead to accelerated wear of components, fatigue failure, increased maintenance requirements, and unexpected breakdowns. These effects not only reduce equipment lifespan but also generate additional operational costs and pose safety risks for personnel and infrastructure. Consequently, understanding and controlling vibrations is a key aspect in the design, operation, and maintenance of mining equipment.

Mathematical modeling represents a fundamental approach for analyzing vibration phenomena and predicting the dynamic behavior of mechanical systems. By formulating models based on physical laws and using differential equations, it becomes possible to describe the motion of vibrating systems and to evaluate the influence of essential parameters such as mass, stiffness, and damping. These models allow engineers to simulate different operating scenarios and to identify critical conditions that may lead to resonance or instability.

In recent years, the development of computational tools and simulation techniques has significantly enhanced the applicability of mathematical modeling in industrial environments. In mining engineering, such approaches are increasingly used to support equipment design, optimize performance, and implement predictive maintenance strategies. Modeling and simulation provide valuable insights into system behavior without the need for costly experimental testing under real operating conditions.

The purpose of this paper is to present the fundamental principles of mathematical modeling applied to vibrations in mining mechanical systems and to analyze their dynamic response under various operating conditions. The study aims to highlight the role of vibration analysis in improving equipment reliability, reducing maintenance costs, and ensuring safe and efficient mining operations.

The analysis of vibrations in mechanical systems is grounded in the principles of classical mechanics, where the dynamic behavior is described using differential equations derived from Newton’s second law. In mining mechanical systems, vibrations are generated by dynamic loads, unbalanced masses, impacts, and interactions between moving components, making their study essential for predicting system response and preventing structural or functional failures.

A fundamental approach to vibration modeling is based on the representation of the system using an equivalent mechanical model composed of mass, stiffness, and damping elements. The simplest case is the single-degree-of-freedom (SDOF) system, which provides a basic yet effective framework for understanding the essential characteristics of vibration phenomena in components of mining equipment. The equation of motion for a damped system subjected to an external excitation force is expressed as:

$$mx(t) + cx(t) + kx(t) = F(t) \qquad (1)$$

Where m represents the equivalent mass, c the damping coefficient, k the stiffness of the system, and x(t) the displacement. This formulation allows the analysis of both free and forced vibrations. In the absence of external forces, the system exhibits free vibration characterized by its natural frequency, while the presence of excitation forces leads to forced vibrations, where the response depends on the relationship between the excitation frequency and the natural frequency of the system. A critical condition occurs when these frequencies coincide, leading to resonance and significantly increased vibration amplitudes.

The dynamic response of the system is strongly influenced by the damping ratio, which governs the rate at which oscillations decay over time. Depending on its value, the system may exhibit underdamped, critically damped, or overdamped behavior, each having direct implications on stability and operational safety. In mining applications, insufficient damping can lead to persistent vibrations, resulting in accelerated wear, reduced component lifespan, and potential mechanical failure.

However, real mining systems are complex and cannot be accurately represented by single-degree-of-freedom models. Equipment such as belt conveyors, crushers, and hoisting systems require multi-degree-of-freedom representations, where multiple masses interact through elastic and damping elements. In practice, modeling must also account for variable loading, structural flexibility, and nonlinear effects. Therefore, while simplified models are useful for initial analysis, advanced simulation methods are often necessary to accurately predict system behavior and support effective vibration control and performance optimization

2. MATERIALS AND METHODS

Belt conveyor systems are among the most frequently used transport installations in mining operations, ensuring the continuous movement of bulk materials under variable loading conditions. Because of their long structure, rotating components, support frames, and non-uniform material flow, these systems are exposed to important dynamic effects. Excessive vibrations may lead to roller wear, belt misalignment, structural fatigue, and a reduction in the overall operational reliability of the installation. For this reason, the study of vibration behavior through mathematical modeling is highly relevant for conveyor system design and maintenance.

In this case study, a simplified section of a mining belt conveyor is analyzed using a single-degree-of-freedom dynamic model. The equivalent system consists of an equivalent mass m, stiffness k, and damping coefficient c, subjected to a harmonic excitation force generated by irregular loading or roller misalignment. The equation of motion is:

$$mx(t) + cx(t) + kx(t) = F_0 \sin(\omega t) \qquad (2)$$

Where x(t) is the vibration displacement, F_0 is the amplitude of the excitation force, and ω is the excitation angular frequency.

For the considered conveyor section, the following equivalent parameters were adopted:

- equivalent mass: m=450 kg
- equivalent stiffness: k=180000 N/m
- damping coefficient: c=2200 Ns/m
- excitation force amplitude: F_0=2500 N

These values are representative of a loaded conveyor segment including belt mass, transported material, and structural compliance of the support frame.

The natural angular frequency of the system is calculated as:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{180000}{450}} = \sqrt{400} = 20 rad/s \qquad (3)$$

The corresponding natural frequency is:

$$f_n = \frac{\omega_n}{2\pi} = \frac{20}{2\pi} = 3,18 Hz \qquad (4)$$

The damping ratio is determined using:

$$\varsigma = \frac{c}{2\sqrt{km}} \tag{5}$$

Thus,

$$\varsigma = \frac{2200}{2\sqrt{180000 \cdot 450}} = \frac{2200}{18000} = 0,122 \tag{6}$$

The obtained value indicates an underdamped system, which is typical for industrial conveyor structures. In such systems, vibrations decay gradually, but resonance can still generate significant amplitude increases. For harmonic excitation, the steady-state amplitude of vibration is given by:

$$X = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \tag{7}$$

To evaluate the system response, four excitation frequencies were considered: 2 Hz, 3 Hz, 3.18 Hz, and 4 Hz.

For f=2 Hz:

$$\omega = 2\pi f = 12,57 rad/s$$

$$X = \frac{2500}{\sqrt{(180000 - 450 \cdot 12,57^2)^2 + (2200 \cdot 12,57)^2}} \tag{8}$$

$$X = 0,02224m = 22,24mm$$

For f=3 Hz:

$$\omega = 18,85 rad/s$$

$$X = \frac{2500}{\sqrt{(180000 - 450 \cdot 18,85^2)^2 + (2200 \cdot 18,85)^2}} \tag{9}$$

$$X = 0,05424m = 54,24mm$$

For f=3.18 Hz, which is approximately equal to the natural frequency, X= 0.05687m=56.87mm and for f=4 Hz, X will be 0.02119m=21.19mm.

Table 1. Calculated vibration amplitudes at different excitation frequency

Excitation frequency (f) [Hz]	Angular frequency (omega) [rad/s]	Amplitude (X) [m]	Amplitude (X) [mm]
2.00	12.57	0.02224	22.24
3.00	18.85	0.05424	54.24
3.18	19.98	0.05687	56.87
4.00	25.13	0.02119	21.19

The results clearly show that the vibration amplitude increases significantly as the excitation frequency approaches the natural frequency of the conveyor system. The maximum displacement, approximately 56.87 mm, is obtained near resonance. This confirms that resonance conditions may severely affect the mechanical behavior of the conveyor, leading to excessive dynamic stresses in rollers, support frames, and the belt itself. An additional useful parameter is the static displacement:

$$X_{st} = \frac{F_0}{k} = \frac{2500}{180000} = 0,01389m = 13,89mm \qquad (10)$$

By comparing this value with the maximum dynamic response, it can be observed that the vibration amplitude near resonance is more than four times higher than the static displacement. This highlights the importance of dynamic analysis, since a system that appears safe under static loading may become critical under periodic excitation.

From an engineering perspective, the obtained results indicate that the operational excitation frequencies of the conveyor system should be kept away from the natural frequency range around 3.18 Hz. In practice, this can be achieved by increasing structural stiffness, adjusting belt tension, improving the alignment of rollers, or introducing additional damping elements. Vibration monitoring systems may also be used to detect abnormal behavior at an early stage and support predictive maintenance strategies.

Therefore, the present case study demonstrates that mathematical modeling provides a reliable method for identifying resonance conditions and estimating vibration amplitudes in mining belt conveyor systems. Even with a simplified model, the analysis offers valuable information for improving reliability, reducing maintenance interventions, and ensuring safer operation in mining environments.

3. CONCLUSIONS

This study emphasized the importance of mathematical modeling in the analysis of vibrations in mining mechanical systems, with a focus on belt conveyor installations commonly used in industrial applications. The approach allowed the representation of the system through simplified dynamic models, providing a clear understanding of its vibration behavior under operational conditions.

The results of the case study showed that vibration amplitude is strongly dependent on the excitation frequency. As the excitation frequency approaches the natural frequency of the system, a significant increase in vibration amplitude occurs, confirming the presence of resonance phenomena, which may have critical effects on system performance.

The analysis also highlighted that the system operates in an underdamped regime, which is typical for mechanical structures used in mining. This behavior allows oscillations to persist and makes the system more sensitive to external dynamic loads, increasing the risk of excessive vibrations.

A comparison between static and dynamic responses revealed that vibration amplitudes under dynamic excitation can significantly exceed those predicted by static analysis. This finding underlines the necessity of including dynamic considerations in the design and evaluation of mining equipment.

From an engineering point of view, the study indicates that resonance conditions should be avoided through appropriate design choices, such as increasing system stiffness, improving damping, and ensuring proper alignment of mechanical components. Additionally, vibration monitoring can play a key role in early fault detection and maintenance planning.

Overall, mathematical modeling proves to be a valuable and practical tool for analyzing and controlling vibration phenomena in mining systems. Its application contributes to improved reliability, reduced maintenance costs, and safer operation, while also supporting future developments in simulation-based engineering approaches.

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